The bit structure of the canonical triad

Exact one-bit descent, restricted two-bit jumps and the KQ funnel geometry

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1 Bit-length structure of the triatrad descent

1.1 Empirical starting point

For the canonical triatrad operator[1]

$$F(N) = N - T_{t(N)}$$

with

$$T_m = \frac{m(m+1)}{2}$$

and the uniquely determined index

$$m = t(N), \qquad m(m-1) \leq N < m(m+1),$$

An empirical study was first carried out to investigate how the binary bit length of an initial value changes throughout the entire descent to a terminal value

$$E(N) \in \{1,2,3,6\}$$

.

To this end, complete descent paths were analysed for starting values with bit lengths of 128, 256, 512, 1024, 2048 and 4096 bits. The analysis comprised 600 complete descent paths with a total of 1026 observed two-bit jumps. The paths examined exhibited strikingly rigid behaviour. The vast majority of all operator applications reduced the bit length by exactly one bit. Among all observed steps involving greater bit loss, only a loss of two bits occurred; a loss of three or more bits was not observed.

The frequency of these two-bit jumps also remained virtually constant across the widely varying initial values. On average, approximately1,7 two-bit jumps occurred per complete descent path. The additional bit losses were concentrated almost entirely in the lower part of the descent paths. Of the observed two-bit jumps,91,7 % occurred at a current bit length of no more than 10 bits and99,8 % at no more than 20 bits. The largest observed two-bit jump occurred at a current bit length of 27 bits.

[1] Meißner, A. (2026). The canonical triatrad of natural numbers. A Vierbaum decomposition and the open problem of terminal class assignment. Zenodo. https://doi.org/10.5281/zenodo.21603457

This resulted empirically in a significantly sharper pattern than a mere asymptotic halving of the initial value. For large sections of a complete descent, the pattern was observed not merely approximately, but exactly at each position

$$b \to b-1 \to b-2 \to b-3 \to \cdots$$

. The few deviations consisted exclusively of individual transitions of the form

$$b \to b-2.$$

The following analysis shows that this behaviour is structurally enforced. Every non-terminal triatrad step loses exactly one or two bits of bit length. Furthermore, the number of two-bit jumps along a complete descent path is globally limited.

1.2 Exact bit loss in a single descent

Let$N$ be a non-terminal positive value and$m = t(N)$. Because

$$m(m-1) \leq N < m(m+1)$$

N can be expressed in the form

$$N = m^2 + d$$

be written as

$$-m \leq d \leq m-1$$

. For the Triatrad successor, the following then holds

$$F(N) = N - \frac{m(m+1)}{2} = \frac{m(m-1)}{2} + d.$$

From this representation, it first follows that

$$F(N) < \frac{N}{2}.$$

Thus,$F(N)$ always has at least one bit fewer than$N$.

The lower bound is given by

$$F(N) - \frac{N}{4} = \frac{m^2 - 2m + 3d}{4}.$$

Since$d \geq -m$, it follows that

$$F(N) - \frac{N}{4} \geq \frac{m(m-5)}{4}.$$

For$m \geq 5$, this therefore holds

$$\frac{N}{4} \leq F(N) < \frac{N}{2}.$$

The remaining small groups can be checked in full immediately. For every non-terminal positive triatrad step, the following therefore holds globally

$$\boxed{1 \leq \mathrm{bitlen}(N) - \mathrm{bitlen}(F(N)) \leq 2.}$$

The bit-length loss of a single step can thus only take the values

$$\boxed{\Delta_{\mathrm{bit}} \in \{1,2\}}$$

. Triples or larger bit-length jumps are ruled out in the Triatrad descent.

1.3 Restriction on two-bit jumps

A two-bit jump can only occur if the current value is sufficiently close to the lower power-of-two boundary of its bit interval. Let

$$b = \mathrm{bitlen}(N)$$

and

$$x = \frac{N}{2^{b-1}}, \qquad 1 \leq x < 2.$$

With the representation

$$N = m^2 + d, \qquad -m \leq d \leq m - 1,$$

the following holds for the Triatrad successor

$$F(N) = \frac{m(m-1)}{2} + d.$$

At the same time, this expression can be written in the form

$$F(N) = \frac{N}{2} - \varepsilon$$

, where

$$\varepsilon = \frac{m-d}{2}.$$

From the constraints on$d$, it follows that

$$\frac{1}{2} \leq \varepsilon \leq m.$$

Since$m = t(N) < \sqrt{N} + 1$ and$N < 2^b$ hold,

$$\varepsilon < 2^{b/2} + 1.$$

For a one-bit step from bit length$b$ to bit length$b-1$, let the normalised successor be

$$x' = \frac{F(N)}{2^{b-2}}.$$

Then the following holds

$$x' = x - \delta_b$$

where

$$\delta_b = \frac{\varepsilon}{2^{b-2}}.$$

This gives the estimate

$$0 < \delta_b < D_b,$$

where

$$D_b = 2^{2-b/2} + 2^{2-b}.$$

A two-bit jump from$b$ to$b-2$ occurs precisely when

$$F(N) < 2^{b-2}.$$

From

$$F(N) = \frac{N}{2} - \varepsilon$$

it follows that the necessary condition is

$$x < 1 + \delta_b < 1 + D_b.$$

A two-bit jump is therefore only possible if the normalised value$x$ lies close to the lower boundary$x = 1$ of its bit interval.

Following a two-bit jump, this situation changes fundamentally. Let$B$ be the bit length of the initial value of such a jump. The subsequent value then has the bit length$B-2$. Its normalisation relative to this new bit interval satisfies

$$x_{B-2} = \frac{F(N)}{2^{B-3}} = 2x_B - 2\delta_B.$$

Since$x_B \geq 1$ and$\delta_B < D_B$, it follows that

$$x_{B-2} > 2 - 2D_B.$$

The successor value thus lies once again in the upper range of its new bit interval.

Suppose that, later on the same descent path, another two-bit jump with an initial bit length of$k \geq 8$ occurs. Between the first two-bit jump at bit length$B$ and this second

two-bit jump, there are only one-bit steps. For the normalised value at bit length$k$, the following therefore holds

$$x_k > 2 - 2D_B - \sum_{j=k+1}^{B-2} D_j.$$

For$B \geq k + 2$, the following holds

$$2D_B + \sum_{j=k+1}^{B-2} D_j < \sum_{j=k+1}^{\infty} D_j.$$

This follows from$D_{B-1} > D_B > 0$ and the fact that all other summands are positive.
If we define

$$S_k = \sum_{j=k+1}^{\infty} D_j,$$

then, by virtue of $k \geq 8$

$$S_k \leq S_8.$$

ForS_8 , we obtain explicitly

$$S_8 = \sum_{j=9}^{\infty} \left(2^{2-j/2} + 2^{2-j}\right) = \frac{17 + 16\sqrt{2}}{64} \approx 0{,}619178.$$

Thus, for a possible second two-bit jump with an output bit length of$k \geq 8$, the following must hold

$$x_k > 2 - S_8 \approx 1{,}380822.$$

On the other hand, a two-bit jump with $k \geq 8$

$$x_k < 1 + D_k \leq 1 + D_8.$$

Because

$$D_8 = \frac{17}{64} = 0{,}265625$$

it follows that

$$x_k < 1{,}265625.$$

The two conditions

$$x_k > 1{,}380822$$

and

$$x_k < 1{,}265625$$

are incompatible. Consequently, on a complete descent path, at most one two-bit jump may occur, the initial value of which has a bit length of at least 8 .
For the remaining finite range

$$1 \leq N < 128$$

all values can be checked in full. The maximum numbers of two-bit jumps within this lower range are:

Terminal value E(N)	maximum number of two-bit jumps	Example
1	2	$17 \to 7 \to 1$
2	2	$33 \to 12 \to 2$
3	2	$34 \to 13 \to 3$
6	1	$16 \to 6$

Table 1

Above this range, at most one further two-bit jump may occur. Consequently, for a complete descent path

$$J(N) \leq 3.$$

For paths with terminal value 6 , the following even holds

$$J(N) \leq 2.$$

The global bound $J(N) \leq 3$ is sharp. The descent path

$$136 \to 58 \to 22 \to 7 \to 1$$

has the bit-length sequence

$$8 \to 6 \to 5 \to 3 \to 1$$

and thus the bit losses

$$2,\ 1,\ 2,\ 2.$$

Thus, three two-bit jumps do indeed occur. The first has the initial bit length 8 , whilst the other two lie within the finite lower range.

1.4 Path length and bit length

Let

$$N_0 = N,\ N_1, \dots, N_{P(N)-1} = E(N)$$

a complete descent path of a non-terminal start value$N$, where$P(N)$ denotes the number of positions, including the start value and the terminal value. The path therefore has

$$P(N) - 1$$

operator applications.

Each transition reduces the bit length by either one or two bits. Let$J(N)$ be the number of two-bit jumps. Then$P(N) - 1 - J(N)$ transitions result in a bit loss of one, and$J(N)$ transitions result in a bit loss of two. The total bit length loss is therefore

$$\begin{aligned} b - \mathrm{bitlen}(E(N)) &= (P(N) - 1 - J(N)) + 2J(N) \\ &= P(N) - 1 + J(N). \end{aligned}$$

Thus, the exact identity holds

$$P(N) = b - \mathrm{bitlen}(E(N)) + 1 - J(N).$$

Equivalently, it follows that

$$b - P(N) = \mathrm{bitlen}(E(N)) - 1 + J(N).$$

The four terminal values have the bit lengths

$$\mathrm{bitlen}(1) = 1,$$

$$\mathrm{bitlen}(2) = \mathrm{bitlen}(3) = 2,$$

and

$$\mathrm{bitlen}(6) = 3.$$

For paths with terminal value1 , the following additionally holds for every non-terminal start value

$$J(N) \geq 1.$$

The equation$F(N) = 1$ has only4 and7 as immediate non-terminal predecessors. Both of these last transitions,

$$4 \to 1$$

and

$$7 \to 1,$$

reduce the bit length by two. Together with $J(N) \leq 3$, the following holds for this class

$$1 \leq b - P(N) \leq 3.$$

For the terminal values 2 and 3 , given their bit length of two and the global bound, the following holds $J(N) \leq 3$

$$1 \leq b - P(N) \leq 4.$$

For the terminal value 6 , the terminal-value-specific bound yields $J(N) \leq 2$

$$2 \leq b - P(N) \leq 4.$$

This yields the global bound for every non-terminal positive initial value, regardless of its terminal class

$$1 \leq b - P(N) \leq 4.$$

Equivalently, the following holds

$$b - 4 \leq P(N) \leq b - 1.$$

The path length of a complete Triatrad descent thus differs by at most four positions from its bit length, irrespective of the size of the starting value. In particular, it follows that

$$P(N) = b + O(1).$$

This statement is stronger than a merely linear asymptotic bound. The complete descent path is not only of the order of the bit length, but deviates from it by a globally bounded additive constant.

1.5 Two-bit jumps for arbitrarily large initial values

The strong concentration of empirically observed two-bit jumps on small bit lengths does not mean that such jumps can only occur within a limited range of numbers. Rather, there exists an infinite family of arbitrarily large initial values for which a two-bit jump occurs immediately.

Let $a \geq 1$

$$N_a = 2^{2a}.$$

Since N_a is a square, we have

$$t(N_a) = 2^a.$$

The Triatrad successor is therefore

$$F(N_a) = 2^{2a} - \frac{2^a(2^a+1)}{2}$$
$$= 2^{2a-1} - 2^{a-1}$$
$$= 2^{a-1}(2^a - 1).$$

For$a \geq 1$, we have

$$2^{2a-2} \leq F(N_a) < 2^{2a-1}.$$

Thus, the initial value has a bit length of

$$\text{bitlen}(N_a) = 2a + 1,$$

whilst for the subsequent value

$$\text{bitlen}(F(N_a)) = 2a - 1$$

. Consequently,

$$\text{bitlen}(N_a) - \text{bitlen}(F(N_a)) = 2.$$

Every value in the infinite family

$$N_a = 2^{2a}$$

thus generates a two-bit jump. Since$a$ can be chosen to be arbitrarily large, two-bit jumps occur for arbitrarily large initial values.

The global constraint therefore does not concern the size of possible two-bit jumps within the number range, but rather their number along a single complete descent path.

2 KQ structure and funnel geometry

2.1 KQ metadata used

For the following structural analysis, only a small portion of the KQ metadata already introduced is required. The complete construction of KQ and KFS is the subject of the previous work on KQ/KFS structures[2] and will not be repeated here.

For a value$N$, let$t_q = t_q(N)$ be the unique KQ group number with

$$t_q(t_q - 1) \leq N < t_q(t_q + 1).$$

where

$$t_p = t_q - 1$$

the group identity is

$$N_0 = t_q(t_q - 1).$$

The position of$N$ within its group is

$$GP = N - N_0.$$

Thus, every value can be written directly in the form

$$\boxed{N = t_q(t_q - 1) + GP}$$

.

For$GP = 0$, the group identity isN_0 . The proper group members have the positions

$$GP = 1, \dots, 2t_q - 1.$$

The centre of the squareGZ is at

$$GP = t_q$$

and thus at

$$N = t_q^2.$$

[2] Meißner, A. (2025). KQ-KFS structures: On the geometric emergence of arithmetic reality. An analysis of canonical quasi-quadratic groups (KQ) and K-focal shell structures (KFS) as structural forms of information-carrying spaces. Zenodo. https://doi.org/10.5281/ZENODO.15683651

For1 $\leq GP < t_q$,N lies on the left-hand side of the groupNL ; for$t_q < GP \leq 2t_q - 1$, it lies on the right-hand side of the groupNR . The number of proper group elements is therefore

$$2t_q - 1,$$

from which the sequence of group widths

$$1,3,5,7,9, \ldots$$

.

The figure atFigure1) illustrates this structure for$N = 1$ through to$N = 31$. The group identitiesN_0 are marked in green and the square centresGZ in red.

N	Tp	Tq	GP	N0	SP	Lage
1	0	1	1	0	0	-
2	1	2	0	2	N0	N0
3	1	2	1	2	-1	NL
4	1	2	2	2	0	GZ
5	1	2	3	2	1	NR
6	2	3	0	6	N0	N0
7	2	3	1	6	-2	NL
8	2	3	2	6	-1	NL
9	2	3	3	6	0	GZ
10	2	3	4	6	1	NR
11	2	3	5	6	2	NR
12	3	4	0	12	N0	N0
13	3	4	1	12	-3	NL
14	3	4	2	12	-2	NL
15	3	4	3	12	-1	NL
16	3	4	4	12	0	GZ
17	3	4	5	12	1	NR
18	3	4	6	12	2	NR
19	3	4	7	12	3	NR
20	4	5	0	20	N0	N0
21	4	5	1	20	-4	NL
22	4	5	2	20	-3	NL
23	4	5	3	20	-2	NL
24	4	5	4	20	-1	NL
25	4	5	5	20	0	GZ
26	4	5	6	20	1	NR
27	4	5	7	20	2	NR
28	4	5	8	20	3	NR
29	4	5	9	20	4	NR
30	5	6	0	30	N0	N0
31	5	6	1	30	-5	NL

Figure1 : KQ groups N = 1 to N = 31

2.2 The reduction in size of the KQ groups

If the KQ structure is examined in descending order of group number, the number of proper group members decreases by two at each transition:

$$2t_q - 1 \rightarrow 2t_q - 3.$$

The square centres

$$1^2, 2^2, 3^2, 4^2, \ldots$$

form the centre of each group. The KQ groups thus possess a natural funnel-like geometry: as the group number decreases, one position disappears on each side, whilst the square centre remains as the unique central position.

This tapering becomes particularly evident in the elementary KQ descent

$$K(N) = N - t_q(N).$$

For the first position on the left of a group, the following applies

$$N_{GP1} = t_q(t_q - 1) + 1.$$

It follows that

$$\begin{aligned} K(N_{GP1}) &= t_q(t_q - 1) + 1 - t_q \\ &= (t_q - 1)^2. \end{aligned}$$

The value$(t_q - 1)^2$ is precisely the centre of the next smaller KQ group. Thus, in general,

$$\boxed{GP1_{t_q} \rightarrow GZ_{t_q - 1}.}$$

Examples include

$$7 \rightarrow 4,$$

$$13 \rightarrow 9,$$

$$21 \rightarrow 16.$$

This illustration is a direct consequence of the group width decreasing by two. As the KQ structure narrows, the outermost left-hand position$GP1$ snaps into place exactly at the centre of the smaller group.

The right-hand side of the group, by contrast, does not exhibit a corresponding edge collapse. If one writes an NR position as

$$GP = t_q + s, \qquad 1 \leq s \leq t_q - 1,$$

this yields

$$K(N) = t_q(t_q - 1) + s.$$

The value thus lies on the left-hand side of the same KQ group. The right-hand side is initially brought back to the left-hand side by the$N - t_q$ descent, whilst the left-hand edge position$GP1$ meets exactly a square centre as the structure narrows further. The funnel effect is therefore not structurally identical on both sides.

3 Connection between KQ and Triatrad

3.1 Triangular numbers as a halved rectangular structure

The Triatrad operator uses the triangular number

$$T_{t_q} = \frac{t_q(t_q + 1)}{2}.$$

The group identity of the following KQ group is

$$N_0(t_q + 1) = t_q(t_q + 1).$$

It therefore follows immediately that

$$\boxed{T_{t_q} = \frac{N_0(t_q + 1)}{2}.}$$

The triangular number subtracted in the triatrad is thus exactly half a rectangular number, which simultaneously defines a KQ group boundary. The descent of the triangular numbers and the KQ group structure are therefore based on the same product of two consecutive natural numbers.

However, no geometric interpretation of this identity is required for the following derivation. What is crucial is that the Triatrad operator can be expressed entirely in the KQ coordinatest_q andGP .

3.2 The triatrad operator in KQ coordinates

With

$$N = t_q(t_q - 1) + GP$$

it follows that

$$\begin{aligned} F(N) &= N - T_{t_q} \\ &= t_q(t_q - 1) + GP - \frac{t_q(t_q + 1)}{2}. \end{aligned}$$

This yields

$$\boxed{F(N) = \frac{t_q(t_q - 3)}{2} + GP.}$$

Equivalently, we can write

$$\boxed{F(N) = T_{t_q - 2} - 1 + GP.}$$

Within a fixed KQ group, the triatrad thus acts as a simple translation of the group positionGP . The order of the KQ positions is preserved. In particular, the following holds for the distinguished boundary position $GP1$

$$F(GP1_{t_q}) = T_{t_q-2}.$$

The following also holds for the square centre of the next smaller group

$$F(GZ_{t_q-1}) = T_{t_q-2}.$$

This yields the identity

$$\boxed{F(GP1_{t_q}) = F(GZ_{t_q-1}).}$$

The positions$GP1_{t_q}$ andGZ_{t_q-1} , which are connected by the KQ funnel, are thus mapped by the triatrad to the same successor value. For the first three cases, this yields

$$F(7) = F(4) = 1,$$

$$F(13) = F(9) = 3,$$

$$F(21) = F(16) = 6.$$

The funnel structure visible in the KQ descent and the Triatrad dynamics are thus directly interlinked algebraically.

3.3 The four terminal values at the bottom of the KQ funnel

A positive value is terminal if the next Triatrad step is no longer positive:

$$F(N) \leq 0.$$

Using the KQ representation, the following condition holds

$$\frac{t_q(t_q - 3)}{2} + GP \leq 0.$$

For$t_q = 1$, this yields the sole positive value

$$N = 1.$$

For$t_q = 2$, exactly the positions$GP = 0$ and$GP = 1$ are possible. They correspond to

$$N = 2, \qquad N = 3.$$

For$t_q = 3$, only$GP = 0$ remains, so

$$N = 6.$$

From$t_q = 4$ onwards, the following already holds

$$\frac{t_q(t_q-3)}{2} > 0.$$

Since$GP \geq 0$ also holds, it follows that for every KQ group with $t_q \geq 4$

$$F(N) > 0.$$

Thus, no further terminal value can occur outside the three lowest KQ groups. The complete terminal set is therefore

$$\boxed{\{1,2,3,6\}.}$$

In KQ coordinates, these four values appear as

N	t_q	GP
1	1	1
2	2	0
3	2	1
6	3	0

Four-terminality is thus not dependent on an empirical search for small terminal values. The KQ funnel geometry restricts possible terminal positions to its lower region from the outset, whilst the KQ form of the triatrad operator, from$t_q = 4$ onwards, excludes any further terminal state.

Since every non-terminal triatrad step generates a smaller positive value, every descending path reaches one of these four values after a finite number of steps. The four terminal values consequently generate the four disjoint backward trees

$$\boxed{\mathbb{N}_{>0} = \mathcal{C}_1 \dot{\cup} \mathcal{C}_2 \dot{\cup} \mathcal{C}_3 \dot{\cup} \mathcal{C}_6.}$$

4 Inverse branching and information structure

4.1 Two regular predecessors

In addition to the forward-directed bit loss, the inverse triatrad structure also possesses a striking binary property. Let$X$ be a positive target value and$N$ an immediate predecessor with

$$F(N) = X.$$

For a possible group index$m = t(N)$, the following holds

$$N = X + T_m.$$

For$N$ to actually belong to the KQ group$m$, the following must

$$m(m-1) \leq X + T_m < m(m+1)$$

the following hold. Substituting$T_m = m(m+1)/2$ yields the equivalent condition

$$\frac{m(m-3)}{2} \leq X < T_m.$$

Let$r$ now be uniquely determined by

$$T_{r-1} \leq X < T_r$$

. Then the two indices

$$m = r$$

and

$$m = r + 1$$

always satisfy the predecessor condition. Thus, every positive triatrad node has at least two immediate predecessors.
A third predecessor with

$$m = r + 2$$

is possible if and only if

$$X = T_r - 1.$$

Hence,

$$\#F^{-1}(X) = \begin{cases} 3, & X = T_r - 1 \text{ für ein } r, \\ 2, & \text{sonst.} \end{cases}$$

The regular inverse triatrad structure is therefore binary. A third branch occurs only at the nodes immediately below a triangular number.

4.2 Rarity of triple branches

The exceptional cases take the form

$$X = T_r - 1.$$

Up to a bound of$M$, there are therefore only approximately as many triple branches as there are triangular numbers below$M$. Their number thus grows only of the order of

$$O(\sqrt{M}).$$

Relative to the$M$ positive integers examined, their proportion is therefore

$$O(M^{-1/2})$$

and vanishes for

$$M \to \infty.$$

The inverse triatrad structure is thus asymptotically a double branch almost everywhere.

4.3 KQ interpretation of backward ambiguity

The KQ representation of the triatrad operator is

$$F(N) = T_{t_q-2} - 1 + GP.$$

If the origin groupt_q is known, the group position can be reconstructed directly from a target value$X = F(N)$:

$$GP = X - T_{t_q-2} + 1.$$

The original value then follows uniquely from

$$N = t_q(t_q - 1) + GP.$$

Within a fixed KQ group, no ambiguity regarding the group position is therefore lost during the triatrad step. The inverse ambiguity arises because the same target value can, in general, originate from two different permissible KQ groups.

For a regular target value, the backward decision therefore consists of choosing between exactly two source groups. This choice can be encoded by a single binary decision:

$$\log_2 2 = 1.$$

At the rare triple junctions, however, a single binary choice is not sufficient.
This statement must be distinguished from a Shannon information loss in the forward step. The bit length of an integer value is not Shannon entropy. The structural correspondence lies in the fact that the forward path loses exactly one binary order of magnitude almost everywhere, whilst the backward path requires exactly one binary choice between two possible source groups almost everywhere.

Thus, two properties of the same operator stand in contrast to one another:

vorwärts: regulär ein Bit Bitlängenverlust

and

rückwärts: regulär zwei mögliche Vorgänger.

The KQ coordinates clarify this second statement: once the origin group has been specified, the predecessor is completely determined.

5 Concluding remarks

The analysis shows that the bit-length dynamics of the canonical triad are considerably more rigid than a mere asymptotic halving would suggest. For every non-terminal step, the following holds exactly

$$\mathrm{bitlen}(N) - \mathrm{bitlen}(F(N)) \in \{1,2\}.$$

A loss of three or more bits in a single step is impossible. Whilst two-bit jumps are possible even for arbitrarily large numbers, their number on a complete descent path is globally bounded:

$$J(N) \leq 3.$$

For the number$P(N)$ of positions in a complete descent path, this yields the exact identity

$$P(N) = b - \mathrm{bitlen}(E(N)) + 1 - J(N)$$

and thus the global bound

$$\boxed{b - 4 \leq P(N) \leq b - 1.}$$

Consequently, the path length differs from the bit length of the starting value by only a finite additive constant, regardless of the size of the starting value:

$$\boxed{P(N) = b + O(1).}$$

The KQ representation provides a supplementary structural perspective on this. With

$$N = t_q(t_q - 1) + GP$$

the Triatrad operator takes the form

$$\boxed{F(N) = \frac{t_q(t_q - 3)}{2} + GP}$$

. Thus, within each KQ group, the operator is a position-preserving translation. At the same time, the tapering of the KQ groups explains the excellent relationship

$$GP1_{t_q} \rightarrow GZ_{t_q - 1},$$

whereby both positions are subsequently mapped by the triatrad to the same subsequent value.

At the lower end of the KQ structure, the condition$F(N) \leq 0$ yields exclusively the four positive values

$$\boxed{1,\ 2,\ 3,\ 6.}$$

For $t_q \geq 4$, a further terminal state is already ruled out by virtue of the KQ form of the operator. The well-known four-tree decomposition of the triatrad thus appears as both an arithmetic and a structural property of the same rectangular number geometry.

The inverse dynamics also exhibit a binary basic structure. Each node regularly has exactly two immediate predecessors; only the values immediately below a triangular number have three. The regular backward decision thus corresponds to a choice between two possible origin groups, whilst the group position within a selected KQ group remains uniquely reconstructible.

The canonical triatrad thus links three closely coupled structures: an exact one-bit descent (bar a few global exceptions), a four-part terminal KQ funnel geometry, and an inverse branching that is binary almost everywhere.